2.9.5 Operation of motor transport (engineering sciences)

DISTRIBUTION OF WEEKLY WORKING HOURS OF TRUCK DRIVERS IN URBAN LOGISTICS

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РАСПРЕДЕЛЕНИЕ ЕЖЕНЕДЕЛЬНОГО РАБОЧЕГО ВРЕМЕНИ ВОДИТЕЛЕЙ ГРУЗОВЫХ АВТОМОБИЛЕЙ В ГОРОДСКОЙ ЛОГИСТИКЕ

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Abstract. The work deals with the transportation of cargo in small batches in an urban environment. It was found that the weekly workload of truck drivers is not always evenly distributed. This is a prerequisite for dissatisfaction among them, as well as a reason for making mistakes or the occurrence of health problems. A partial integer nonlinear mathematical model is proposed to solve the problem. The solution of the task allows the difference between the driver who has worked the longest and the one who has worked the least to be minimal and the workload of the drivers even.

We offer two possible solutions of this task (the example is based on a real situation from practice). In the first case it is the rotation principle, when three interchangeable workers working three days on three different activities. In the second case, seven workers performing six activities in one week, it is precisely determined which worker must perform activities. As a result of the calculations, the option, where the difference between the minimum and maximum time is 1.7 hours for the five working days, because it varies from 33.7 hours to 35.4 hours. In this variant, the rotation principle can also be applied in order for workers to be loaded equally. These options do not exclude the search for other solutions satisfying the problem.

Аннотация. Рассматриваются перевозки грузов небольшими партиями в городских условиях. Выявлено, что недельная нагрузка водителей грузовых автомобилей не всегда распределяется равномерно. Это становится причиной неудовлетворенности среди водителей, а также может привести к совершению ошибок или возникновению проблем со здоровьем. Для решения проблемы предлагается частично целочисленная нелинейная математическая модель. Ее применение позволяет сделать разницу между временем, затраченным водителем, проработавшим больше всех, и водителем, проработавшим меньше всех, минимальной, а нагрузку на водителей – равномерной. Предлагается два возможных варианта решения этой задачи (пример основан на реальной ситуации из практики). В первом случае это принцип ротации, при котором трое взаимозаменяемых рабочих в течение трех дней меняют виды работ. Во втором случае семь рабочих выполняют шесть видов работ в течение одной недели, при этом точно определено, кто из рабочих какой вид работ должен выполнять. В результате расчетов было найдено решение, при котором разница между минимальным и максимальным временем составляет 1.7 часа за пять рабочих дней (время работы варьируется от 33.7 до 35.4 ч). Здесь также может быть применен принцип ротации, чтобы рабочие были загружены равномерно. Эти варианты не исключают поиска других решений, удовлетворяющих поставленной задаче.

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Key words: truck driver, nonlinear mathematical model, transport optimization

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Introduction

The world's population is becoming increasingly concentrated in cities and the urban population is expected to grow from 3.6 billion in 2011 to 6.3 billion by 2050 [1]. The increase in the number of people in the cities leads to an increase in motor vehicles, air pollution, and congestion in the streets, which makes it difficult for drivers who have to perform deliveries on predetermined schedules related to the serviced sites.

This requires optimization of the routes on which the transport is performed and selection of ecological transport means. On the issues of the route optimization and cost reduction in [2] solutions have been proposed that reach almost 30 % savings. With regard to urban pollution, a model has been proposed in [3] for Belgrade, the capital of Serbia, which uses a limited number of electric vehicles and complementary motor vehicles with internal combustion engines to solve this problem.

The distribution of goods to commercial sites in cities has a number of specific features such as: delivery of goods in smaller volumes; sensitivity to the delivery time interval; recurrence of supplies for the day to replenish stocks, due to the small size of most stores in the central parts of the city; repetitive deliveries of certain food products due to the outlet customers' requirement for them to be fresh (e.g. bread and bakery products).

This is a prerequisite for the need of organizing a large number of delivery, collection or delivery/collection routes (a special case of circular routes) for the transport of goods in small batches [4]. Each of these routes has significant differences in the time for one revolution of the car. Thus, the duration of work for the fulfilment of the order varies for the drivers, which is why some of them are busier, others are dissatisfied, especially if the pay is the same or with bonuses, but also due to difficult driving conditions in cities associated with the chance of traffic accidents and injuries that further increases this stress.

Issues of the impact of fatigue on drivers are discussed in [5]. It is determined that the most common causes of fatigue and dissatisfaction among drivers are related to the management of working hours or opportunities for work and rest. With regard to research on the hazards most strongly associated with driver injuries in [6], it is suggested that risk management efforts in the industry should focus on the organisation of work and vehicle-related hazards, and more specifically, to work practices that cause frequent stress among drivers. Another important reason given by the authors who conducted research in Norway is the speed with which vehicles are driven on road sections under repair with and without workers on them, as well as the influence of road signs [7]. Of the 815 drivers surveyed, it was found that the presence of workers in the section leads to a reduction in the speed of vehicles, in contrast to the cases where there are no workers, due to the risk taken by drivers and their perception of danger.

One of the possible solutions of the above is optimization for the even load of drivers in terms of driving time and downtime for loading and unloading operations throughout the working week, provided that the time difference between the worker who has worked for the longest period of time and the one who has worked the shortest time is as little as possible.

Results

General characteristics of the routes for transporting goods in small batches and organization of the drivers' work

Delivery of goods in small batches can be done at your own expense or for a fee. In practice, most often this delivery is made for a fee by licensed suppliers or by people who offer their services when the delivery coincides with their route, known as crowd-shipping [8, 9]. The disadvantage of crowd-shipping is that the issue of the responsibility of the provider and the legality of the service in different countries is not well clarified. In this regard, this paper will consider shipments performed by licensed suppliers.

The following types of routes are considered in the literature: delivery, collection and delivery-collection [10]. In the case of a delivery route, the loaded rolling stock at the initial point transports the consignment in batches to the points along the route until it is gradually unloaded. In the collection route, the rolling stock, passing successively through the loading points, is gradually loaded and the cargo is transported to one point (Fig. 1).

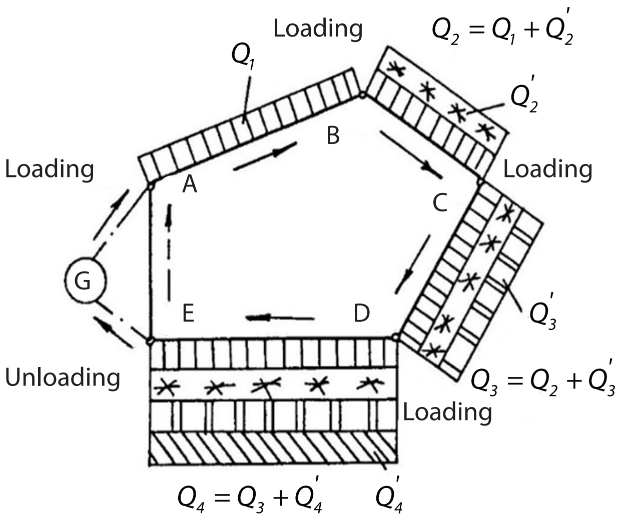

Fig. 1. Diagram of a collection route
Рис. 1. Схема сборного маршрута

Figure 1 shows the route where the vehicle starts from the garage (G) and goes to the first point (A), where the amount of cargo Q_1 is loaded, which is transported to the second point (B). Then, at point (B) is loaded $Q_2^{'}$ and then the total amount of load in the vehicle becomes Q_2. The scheme is repeated in the following points, where in point (C) the quantity of cargo loaded is $Q_3^{'}$, and up to the last point in the case (D), where $Q_4^{'}$ is loaded. The total quantity of cargo is delivered in point (E). From point (E) to point (A), the vehicle returns empty, without any load and so this route can be repeated until the work is completed. If there is no other work, then from point (E), the vehicle returns to the garage (G).

In the case of collection and delivery routes, the rolling stock simultaneously collects one load and carries another (for example, the delivery of food products with the simultaneous collection of packaging). The following indicators are defined for these routes:

- turnover length for delivery – l_0 . For collection routes it is equal to route length $l_0 = l_M$;
- turnover time – t_0 .

$$t_0 = \frac{l_0}{V_T} + t_{T-R}, \text{ h}, \tag{1}$$

where V_T is the average technical speed of the rolling stock, km / h;

t_{T-R} – the average time for loading and unloading for one course, h.

For collection/delivery routes:

$$t_{T-R} = \sum_{j=1}^{k} t_{T-R_j}, \text{ h}, \tag{2}$$

where t_{T-R_j} is the loading/unloading time at the j^{th} point of the route, h;

k – the number of points on the route.

Given the planned work, let p routes be organized for each day of the week with a turnaround time of $t_{01}, t_{02}, ..., t_{0p}$ and a turnaround time of no more than 8 h, i.e. $t_0 \le 8\,h$ (Fig. 2).

Moreover $t_{01} \ne or = t_{02} \ne or = \ldots t_{0p}$.

The organization of drivers' working hours in transport requires careful planning. This is the time from the beginning to the end of the work

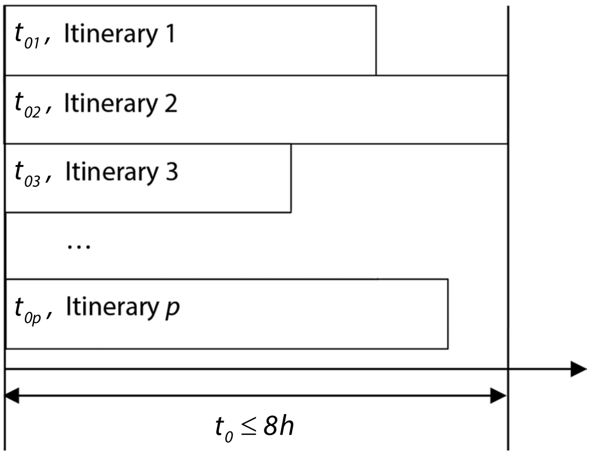

Fig. 2. Period of time for work on the route
Рис. 2. Период времени работы на маршруте

assignment during which the driver is available to the employer and exercises his functions or activities. Conditionally, these activities can be divided into two main groups. The first group includes driving times and loading and unloading times, and the second group includes cleaning, maintenance and any other activities to ensure the safety of the vehicle and its load, or activities to comply with legal or regulatory obligations, as well as appropriate assignment of other tasks.

The first group of activities is in periods related to the times in the car's duty order.

The balance of the total working time T_w for execution of the duty order for one vehicle is:

$$T_w = t_{run} + t_{T-R} + t_P, \tag{3}$$

where $t_{run} = \dfrac{L}{V_t}$ is the travel time, h, at distance *L*, km, for the time of the duty order and technical speed V_t, km/h;

t_{T-R} – the total loading and unloading time of the vehicle, h;

t_P – the idle time spent for technical or organizational reasons, h.

For each unit of rolling stock, the vehicle hours (AT_w) in a work day are the sum of all hours of its stay in the line for a given period of time D_w days, determined by the dependence:

$$AT_w = \sum_{i=1}^{D_w} T_{w_i}, \text{ h}, \tag{4}$$

where T_{w_i} are the hours of operation of the one vehicle on the line on the i^{th} day, where $i = \overline{1, D_w}$.

For a group of vehicles, or for the entire fleet, the car hours are $AT_w^{'}$, in duty order:

$$AT_w^{'} = \sum_{j=1}^{A_n} \sum_{i=1}^{D_w} T_{w_{ij}}, \tag{5}$$

where $T_{w_{ij}}$ is the time in duty order of the j^{th} rolling stock on the i^{th} day of the period D_w, A_n is the number of cars.

Mathematical model of the problem for even distribution of total weekly working hours of drivers

The problem of providing even workload for all drivers within one working week is multivariate and requires special methods and means for rational distribution of drivers' work. In order to draw up a model of the problem of evenly distributing the total weekly working hours between drivers, the case is considered in which for p consecutive days (five working days), the same type of activities ("work") must be carried out every day, each of which with a duration t_j, $j = \overline{1, n}$. The everyday activities can be performed by m employees. Each activity must be carried out by one worker every day (one activity cannot be carried out on the same day by two or more workers). A worker cannot take on more than one activity per day, i.e. $m \geq n$.

The goal is as follows: for a period of *p* days, each worker has worked approximately the same time (all days). Given the nature of the organisation of transport, the probability of achieving this is very small. In order to achieve the formulated goal, a criterion is applied that the time difference between the worker who has worked the longest time and the worker who has worked the shortest time is as small as possible. Decision-making with a clear single criterion (single-criteria tasks) is relatively easy, but with more than two criteria (two goals, multi-criteria tasks [11–13] the decision is complex and sometimes purely subjective factors have to be included to find a solution). Binary variables are involved in the composition of such models, and the following notations are introduced to compile the model:

m is number of workers;

n – number of activities (works);

p – number of days on which these activities are carried out each day;

$$x_{ijk} = \begin{cases} 1, \text{ if } i^{th} \text{ worker performs } j^{th} \text{ activity in } k^{th} \text{ day} \\ 0, \quad \text{otherwise}, \quad i = \overline{1,m},\ j = \overline{1,n},\ k = \overline{1,p} \end{cases};$$

t_j – time to complete an j^{th} activity, $j = \overline{1,n}$.

We are looking for the unknowns x_{ijk} (binary variables) so:

$$Z = min_x \left\{ max_i \left(\sum_{j=1}^{n} \sum_{k=1}^{p} t_j x_{ijk} \right) - \right.$$
$$\left. - min_i \left(\sum_{j=1}^{n} \sum_{k=1}^{p} t_j x_{ijk} \right) \right\}; \tag{6}$$

$$\sum_{i=1}^{m} x_{ijk} = 1,\ \forall\, j = \overline{1,n},\ \forall\, k = \overline{1,p}; \tag{7}$$

$$\sum_{j=1}^{n} x_{ijk} \le 1,\ \forall\, i = \overline{1,m},\ \forall k = \overline{1,p}; \tag{8}$$

$$x_{ijk} \in \{0,1\},\ i = \overline{1,m},\ j = \overline{1,n},\ k = \overline{1,p}, \tag{9}$$

where x indicates a vector (column) with all unknown search values x_{ijk} "arranged in a linear manner". Thus, a matrix x_{ijk}, is three-dimensional and contains m rows, n columns, and the depth is p. If all elements of are assigned with sequential numbers – starting with the elements from column 1 of the first 2D matrix, then – the elements from column 2 of the first 2D matrix, etc., and when all the columns from the first 2D matrix are numbered in depth, then the same sequential numbering continues in the next 2D matrix. This is how the expression "arranged in a linear manner" should be interpreted.

Let T_i indicate the total work of a i^{th} worker, and let T be a vector (column, shown in formula (11)) with these values. Then:

$$T_i = \sum_{j=1}^{n} \sum_{k=1}^{p} t_j x_{ijk}; \tag{10}$$

$$T = \begin{bmatrix} T_1, \\ T_2, \\ \dots, \\ T_m \end{bmatrix}, \tag{11}$$

where $max_i\left(\sum_{j=1}^{n} \sum_{k=1}^{p} t_j x_{ijk}\right) = max(T_i)$ – the work done by the worker who worked the most hours; $min_i\left(\sum_{j=1}^{n} \sum_{k=1}^{p} t_j x_{ijk}\right) = min(T_i)$ – the work done by the worker who worked the least hours.

t will indicate a vector column with a time value element t_j. Condition (6) is the objective function with which to minimise the difference between the times of the driver who will work the longest time with that of the driver who will work the shortest time for a given period. With the notations (6) thus introduced, it can also be expressed in the following way:

$$Z = min_x \left\{ max_i T - min_i T \right\}. \tag{12}$$

Condition (7) corresponds to the fact that each activity on all days must be performed by only one driver.

Condition (8) reflects the fact that on all days, one driver can perform one activity per day at most. A driver may not be employed for a given day (if $m > n$, then $m - n$ is the number of workers per day who are not employed).

Condition (9) is a condition for binary of the variables due to the specifics of the model.

Thus, conditions (6) – (9) represent an optimization mathematical model of the problem posed with objective function (6) and constraints (7) – (9).

The calculations and programs are made using the product MATLAB version R2017b, which is one of the most powerful calculation software tools [14]; these are the so-called CAS systems (Computer Algebra System). MATLAB has a rich palette of optimization tools – linear, nonlinear, one-dimensional, multidimensional, continuous, integer, partially integer, single-criteria, multi-criteria, etc. In MATLAB the embedded optimization functions also have a wide choice of settings, depending on the specific problems [14]. In addition, MATLAB also supports its own programming language, which can be used to make solvers for solving problem (6) – (9) at random data entry m, t and p.

The model (6) – (9) is an integer problem of class NP complete problems (nondeterministic polynomial time). Such tasks are extremely labor intensive as they take up a lot of time and memory resources. For their complete solution, it is sometimes necessary to completely crawl the tree of all possible options, and these options, even with relatively small dimensions, can reach a colossal number of possible options for verification. In the specific problem (6) – (9), this number is 2^{mnp}. If we consider the work of 10 drivers working on 8 routes per day, then for 7 days, the total number of possible solutions is $2^{10\cdot 8\cdot 7} = 3.774\cdot 10^{168}$, which means that if a software makes one crawl of the decision tree for one millionth of a second, it would take it about $\frac{2^{10\cdot 8\cdot 7}}{60\cdot 60\cdot 24\cdot 365.4\cdot 1000000} \approx 1.1954\cdot 10^{155}$ years to complete all the solutions. This requires the use of approximate methods: heuristic, stochastic [15], genetic, and other algorithms [16–18]. With these methods, solutions are obtained that are close to optimal.

The especially built-in ga function of MATLAB makes it possible to solve problems in which it is possible that both the objective and the constraints are nonlinear [13, 14]. This solver is based on genetic algorithms [18–20]. Table 1 shows the main differences between classical and genetic algorithms.

Table 1
Таблица 1

Main differences between classical and genetic algorithms
Основные различия между классическим и генетическим алгоритмами

Classical algorithms	**Genetic algorithms**
Generate a single point on each iteration. The sequence of points tends to the optimal solution	Generate a population of points on each iteration. The point of the population with the lowest value of fitness function tends to the optimal solution
The choice of the next point is determined	The choice of the next population is random

The solver ga solves the problem to a minimum, i.e. it minimizes the expression:

$$max_i T - min_i T, \tag{13}$$

or the expression:

$$max_i\left[T_1, T_2, \ldots, T_i, \ldots T_m\right] - \\ - min_i\left[T_1, T_2, \ldots, T_i, \ldots T_m\right]. \tag{14}$$

The two components (minuend, subtrahend) of (13) are functions that are in the most general case non-differentiable and discrete in the domain. These assumptions would make it difficult for almost any modern solver, given that it initially seeks to calculate the gradient of the target function. To avoid this, the functions (14) are expressed by elementary functions and a finite number of arithmetic operations applied to them (including rising to an arbitrary degree). Thus, the non-differentiable functions $max_i\left(f_1, f_2, \ldots f_q\right)$ and $min_i\left(f_1, f_2, \ldots f_q\right)$, will be subject to differentiation, provided that their arguments are continuous, as well as the functions f_i themselves. In this case, the functions f_i are continuous, but the arguments are discrete.

Considering the displayed border crossings (15) and (16):

1) if $f_i \geq 0,\ r \in \mathbb{R}$, for $\forall i = \overline{1,q}$, then

$$\lim_{r\to\infty}\left(f_1^r + f_2^r + \ldots + f_q^r\right)^{\frac{1}{r}} = max_i f_i, \tag{15}$$

2) if $f_i \geq 0,\ r \in \mathbb{R}, \forall i = \overline{1,q}$, then

$$\lim_{r\to\infty}\left(f_1^{-r} + f_2^{-r} + \ldots + f_q^{-r}\right)^{-\frac{1}{r}} = min_i f_i, \tag{16}$$

the objective function (6) can be represented as

$$Z = min_x\left\{\lim_{r\to\infty}\left(T_1^r + T_2^r + \ldots + T_m^r\right)^{\frac{1}{r}} - \right. \\ \left. - \lim_{r\to\infty}\left(T_1^{-r} + T_2^{-r} + \ldots + T_m^{-r}\right)^{-\frac{1}{r}}\right\}. \tag{17}$$

Expressions (15) and (16) are differentiated (with continuous arguments).

In numerical calculations, this boundary transition would take an extremely long time for each iteration. Therefore, a sufficiently large, specific value is set for *r*. Numerical experiments on test tasks show that it is enough for the parameter r to take values around 30-40, but for this research the value of the parameter has been increased to 100.

Table 2 shows three script files programmed in the MATLAB R2017b environment, solving problem (6) – (9), using the genetic solver ga [19, 20]. For their use, they are in a separate folder, which is "selected" by the user for the current one. This is due to the use of global variables (global variables are listed in the presented script files and functions) in the MATLAB environment. The input variables used in the calculations are number of workers *m*, a vector with values of the times of activities *t* and the number of days *p*.

Table 2
Таблица 2
Script files programmed in MATLAB R2017b environment, solving problem (6) – (9), using the genetic solver ga
Файлы сценариев, запрограммированные в среде MATLAB R2017b, решающие задачу (6) – (9) с использованием генетического алгоритма

File 1 realizing target function (6)	File 2 realizing target function (17), at $r = 100$	File 3 (named optiProblems – script file) – this is the main file. In it, the user enters the values of the number of workers *m*, a vector with values of the times of activities *t* and the number of days *p*. Then the calculation is started with the Run button (or the optiProblems command is set in the command window). In the specific example, the parameters are as follows: $m = 10;\ t = [1, 2, 3, 4, 5, 6, 7, 8];\ p = 7$

In the *optiProblems* script file which solves problem (6) – (9), in the ga server a reference is made to the target function. This function can be set in type (6) or in type (17) to achieve more accurate and faster calculations. In the *optiProblems* script, if the server ga is called with the first function – *myfun1*, then the optimization will be performed with the target function in the form (6), if it is called with the second – *myfun2*, then the optimization will be performed with the target function in the form (17). The output data are *p* number of matrices, each matrix being responsible for the distribution of activities in one working day. The rows in the matrix correspond to the workers and the columns – to the activities. If an element in the matrix assumes a value of 1, then the worker who corresponds to the order of the matrix for this element performs the activity corresponding to the activity in the given column for this element. The data output is also the value of the objective function (non-negative number, at best equal to zero).

Results of application of the mathematical model for determining the work of drivers

In order to evaluate the performance of the already built solver optiProblems, test variants of real tasks have been considered. In this case, two are provided. In the first option, 3 workers are considered who perform 3 activities for 3 days. The duration of the activities is indicated, respectively, by 1, 2, 3. The input data of the program are: $m = 3;\ t = [1, 2, 3];\ p = 3.$

The results are as follows:

$$T(:,:,1) = \begin{pmatrix} 0 & 0 & \mathbf{1} \\ 0 & \mathbf{1} & 0 \\ \mathbf{1} & 0 & 0 \end{pmatrix},\ T(:,:,2) = \begin{pmatrix} \mathbf{1} & 0 & 0 \\ 0 & 0 & \mathbf{1} \\ 0 & \mathbf{1} & 0 \end{pmatrix},$$

$$T(:,:,3) = \begin{pmatrix} 0 & \mathbf{1} & 0 \\ \mathbf{1} & 0 & 0 \\ 0 & 0 & \mathbf{1} \end{pmatrix}.$$

The result of the first matrix (T(:,:,1)) shows that the first worker (row one) on the first day will perform a third activity (column three, bold unit), the second worker (row two, bold unit) on the first day will engage in a second activity (column two, bold unit), and the third (row three) – on the first day will perform a first activity (column one). The interpretation of the other matrices is similar. They are for the next two days. The value of the objective function (TZ) is 0, and the problem has a trivially optimal solution. One such solution would be (on a rotating basis) if the first worker performed the first activity on the first day, the second day engaged in the second and the third day the third activity. The ordinance of these activities by days for the first worker would look like (1, 2, 3). The ordinance of the activities for the second worker by days (2, 3, 1) and for the third worker (3, 1, 2). In this way, each worker performed each activity once and the target function has a value of 0 (in the result, this is the value of TZ).

This is a test option and was chosen to have a trivial solution – one such optimal solution is through a cyclic change, i.e. worker 1 performs activity 1 during the first day, activity 2 during the second and activity 3 during the third day. Worker 2 performs activity 2 on day 1, activity 3 on day 2 and activity 1 on day 3. Worker 3 performs activity 3 on day 1, activity 1 on day 2 and activity 2 on day 3.

In the next variant 7 workers for 5 days have to perform 6 activities with duration in hours respectively: 6.6, 7.4, 8, 8.3, 8.7 and 9 hours. The input data are: $m = 7; t = [6.6, 7.4, 8, 8.3, 8.7, 9]; p = 5$. For convenience, the solution is given in Table 3.

Table 3
Таблица 3

Number of an activity carried out by a worker on a given day
Работа, выполняемая сотрудниками в определенные дни

Day / Worker	Day 1	Day 2	Day 3	Day 4	Day 5
Worker 1	**2**	**1**	**1**	**1**	**3**
Worker 2	5	0	6	3	4
Worker 3	1	2	2	2	1
Worker 4	4	5	4	0	6
Worker 5	6	6	0	4	2
Worker 6	3	3	5	6	0
Worker 7	0	4	3	5	5

The results in Table 3 mean the following: Worker 1, on the first day performs activity 2 (in bold), on the second, third, and fourth day activity 1 (in bold), and on the fifth – activity 3 (in bold).

Zero in a given row and column means that the respective worker on a given day is not performing any activity and is resting (in grey boxes). For example, Worker 5 does not work on Day 3. The working time of each employee during the week is as follows: Worker 1 – works 35.2 hours; Worker 2 – works 34 hours; Worker 3 – works 35.4 hours; Worker 4 – works 34.3 hours; Worker 5 – works 33.7 hours; Worker 6 – works 33.7 hours and Worker 7 – works 33.7 hours.

The maximum time of work during the week of Worker 1 is 35.4 hours, and the minimum of workers 5, 6 and 7, which is 33.7 hours. Thus, the difference between the minimum and maximum time is 1.7 hours for the five working days. This example is based on a real situation from practice and no optimal solution is known for it. Without an algorithm based on complete exhaustion, this solution can't be claimed to be the optimal. The solution obtained here on the basis of genetic algorithms is completely satisfactory from a practical point of view. If for the person making the decision (the manager of the company), this decision is not satisfactory, the search for the next solution should be continued, with the idea to find a better solution if there is one.

Conclusion

The paper presents a solution to a problem in which such a distribution of workers' time during the week is sought, where the difference between the time of the worker who has worked the longest hours and the worker with the shortest hours is minimal. For this purpose, a mathematical model has been created which is an integer problem of class NP complete problems (nondeterministic polynomial time). The solution of the model is done by using the built-in ga function of MATLAB, based on genetic algorithms, which allows solving problems in which both the target function and the constraints may be nonlinear. For this purpose, script files have been created, programmed in the MATLAB R2017b environment, solving the task with the help of the genetic ga solver.

The results obtained in the two tests, which seek for the difference between the minimum and maximum working time of workers to be minimal, lead to the following:

In the first case with three workers working three days on three different activities, the most appropriate option is the rotation principle, in which each worker changes his job every day/week, if possible, and the workers are interchangeable.

In the second case with seven workers performing six activities in one week, it is precisely determined which worker must perform which activities, where the difference between the minimum and maximum time is 1.7 hours for the five working days, because it varies from 33.7 hours to 35.4 hours. In this variant, the rotation principle can also be applied in order for workers to be loaded equally. If for the person making the decision (the manager of the company), this decision is not satisfactory, the search for the next solution should be continued until a better solution is found.

The proposed model is suitable in cases where workers are dissatisfied with the fact that they work different amount of time, with uneven workload, are interchangeable, and receive equal remuneration or bonuses for most of the work performed.

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